<table>
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<th>Standards</th>
<th>Lab and Demo Planning</th>
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<tr>
<td><strong>Chapter Opener</strong></td>
<td>See page 14T for a key to the standards.</td>
<td></td>
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<tr>
<td><strong>Section 8.1</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| 1. **Describe** angular displacement. | UCP.2, UCP.3, A.1, A.2, B.4 | **Student Lab:**  
Launch Lab, p. 197: objects that roll, meterstick, foam or wood board  
**Teacher Demonstration:**  
Quick Demo, p. 199: bicycle wheel, meterstick, cloth measuring tape |
| 2. **Calculate** angular velocity. | | |
| 3. **Calculate** angular acceleration. | | |
| 4. **Solve** problems involving rotational motion. | | |
| **Section 8.2** | | |
| 5. **Describe** torque and the factors that determine it. | UCP.2, UCP.3, B.4 | **Student Lab:**  
Additional Mini Lab, p. 205: pivoting balance beam, one 50-g mass, one 100-g mass, and one 200-g mass.  
**Teacher Demonstration:**  
Quick Demo, p. 208: two 1-m lengths of 2-cm PVC water pipe, four 0.3-m lengths of steel reinforcing rods (rebar) |
| 6. **Calculate** net torque. | | |
| 7. **Calculate** the moment of inertia. | | |
| **Section 8.3** | | |
| 8. **Define** center of mass. | UCP.1, UCP.2, UCP.3, A.1, A.2, B.4, E.1, F.5 | **Student Lab:**  
Mini Lab, p. 213: pencil, cardboard (8.5×11 piece), scissors  
**Additional Mini Lab**, p. 216: stool that rotates easily; 2×10 lumber, 2.5 m long; heavy clamps; ball  
Physics Lab, pp. 218–219: meterstick, two 5-N spring scales, two ring stands, two burette clamps, 500-g hooked mass, 200-g hooked mass  
**Teacher Demonstration:**  
Quick Demo, p. 212: large piece of foam (10 cm thick, about 30×50 cm), two chemiluminescent sticks (about 15 cm long)  
Quick Demo, p. 213: none |
| 9. **Explain** how the location of the center of mass affects the stability of an object. | | |
| 10. **Define** the conditions for equilibrium. | | |
| 11. **Describe** how rotating frames of reference give rise to apparent forces. | | |

**Differentiated Instruction**

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<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>Level 1 activities should be appropriate for students with learning difficulties.</td>
</tr>
<tr>
<td>L2</td>
<td>Level 2 activities should be within the ability range of all students.</td>
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| **FAST FILE** Chapters 6–10 Resources, Chapter 8  
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  Transparency 8-2 Master, p. 95  
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  Reinforcement, p. 89  
  Enrichment, pp. 91–92  
  Section 8-1 Quiz, p. 85  
  Teaching Transparency 8-1  
  Teaching Transparency 8-2  
  Connecting Math to Physics |  
  **TeacherWorks™** includes: Interactive Teacher Edition  
  Lesson Planner with Calendar  
  Access to all Blacklines  
  Correlation to Standards  
  Web links  
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  Section 8.1 Presentation  
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  Transparency 8-4 Master, p. 99  
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  Teaching Transparency 8-4  
  Connecting Math to Physics |  
  Interactive Chalkboard CD-ROM:  
  Section 8.2 Presentation  
  **TeacherWorks™** CD-ROM |
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  Mini Lab Worksheet, p. 73  
  Physics Lab Worksheet, pp. 75–78  
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  Interactive Chalkboard CD-ROM:  
  Section 8.3 Presentation  
  **TeacherWorks™** CD-ROM  
  Problem of the Week at physicspp.com |

**Assessment Resources**

| FAST FILE** Chapters 6–10 Resources, Chapter 8  
  Chapter Assessment, pp. 103–108  
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  Physics Test Prep, pp. 15–16  
  Pre-AP/Critical Thinking, pp. 15–16  
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  Interactive Chalkboard CD-ROM:  
  Chapter 8 Assessment  
  **ExamView®** Pro Testmaker CD-ROM  
  Vocabulary PuzzleMaker  
  **TeacherWorks™** CD-ROM  
  physicspp.com |
Chapter Overview
Like linear motion, rotational motion obeys Newton’s laws. But there is an added twist: different parts of an object being rotated experience different velocities and accelerations. New concepts, like torque and moment of inertia, and several new symbols are required to describe this common type of motion. Torque is also needed to determine whether an object is in rotational equilibrium.

Think About This
Amusement-park rides are designed to accelerate the rider in as many ways as possible. See page 217.

Key Terms
radian, p. 197
angular displacement, p. 198
angular velocity, p. 198
angular acceleration, p. 199
lever arm, p. 201
torque, p. 202
moment of inertia, p. 205
Newton’s second law for rotational motion, p. 208
center of mass, p. 211
centrifugal “force,” p. 216
Coriolis “force,” p. 217

Think About This
Why do people who ride amusement-park rides that spin in circles, such as this one, experience such strong physical reactions?

Purpose
to explore the acceleration of objects that have different moments of inertia

Materials
objects that roll—soup can with ends removed, solid ball, large-diameter wood dowel (solid cylinder) or can of cream of celery soup; meterstick; foam board or similar-sized wood board

Teaching Strategies
Try to have objects of the same diameter. Be sure the can does not have sharp edges. Students can substitute or add any object that rolls down the incline without slipping. Also try objects of the same shape and mode of rotation but with different mass or length.
1 FOCUS
Bellringer Activity
Rotating Football Using a football, first ask students when it can be treated as a point particle. When it is far enough away that its size isn’t an important factor. As you hold it in your hands and casually rotate it in all possible orientations, ask them to brainstorm all the quantities that would have to be determined to describe where the football is, how to orient it, and how it is moving. Quantities should include not only \( x, y, \) and \( z \) (and the three velocities) for the center of the ball, but also the ball’s orientation about three rotational axes (one vertical and two horizontal) and how fast it is rotating about those three axes. Briefly discuss other objects whose rotations are important to them. ▶ Spatial-Visual

Tie to Prior Knowledge
Linear Motion Students will use the quantities that describe linear motion (position, velocity, and acceleration) and the geometry of the circle to develop equations for rotational motion.

8.1 Describing Rotational Motion

You probably have observed a spinning object many times. How would you measure such an object’s rotation? Find a circular object, such as a CD. Mark one point on the edge of the CD so that you can keep track of its position. Rotate the CD to the left (counterclockwise), and as you do so, watch the location of the mark. When the mark returns to its original position, the CD has made one complete revolution. How can you measure a fraction of one revolution? It can be measured in several different ways. A grad is \( \frac{1}{400} \) of a revolution, whereas a degree is \( \frac{1}{360} \) of a revolution. In mathematics and physics, yet another form of measurement is used to describe fractions of revolutions. In one revolution, a point on the edge travels a distance equal to \( 2\pi \) times the radius of the object. For this reason, the radian is defined as \( \frac{1}{2}\pi \) of a revolution. In other words, one complete revolution is equal to \( 2\pi \) radians. A radian is abbreviated “rad.”

Expected Results The ball will accelerate fastest, then the solid cylinder, and then the hollow can.

Analysis Balls should always beat solid cylinders, which will always beat hollow cans.

Critical Thinking The distribution of mass determines the acceleration of a rolling object down a ramp. Recall that the acceleration of a falling object does not depend on its mass; the same is true of a rolling object. To demonstrate, tape two identical cans together and race the joint cans against a single can. The torque on the ball and two cans depends on the radius and the tilt of the board. The farther the mass is from the axis of rotation, the greater the moment of inertia, and the smaller the angular acceleration. See Table 8–2 for the moment of inertia of common objects.
2 TEACH

Concept Development

■ Greek Symbols The symbols used for the quantities involved in rotational motion, \( \theta \) (theta), \( \omega \) (omega), \( \alpha \) (alpha), and \( \tau \) (tau) will be unfamiliar to most students. Point out that these symbols will be used to help distinguish between linear and rotational motion.

■ Radians Angle turned in radians (rad) is based on the ratio between the arc length and the radius of a circle. Point out that in calculations, radian units are really “dimensionless.”

Reinforcement

Degrees and Radians To help students become accustomed to radian measure, create a pie diagram showing a circle, frequently used angles (30°, 45°, 60°, 90°, 120°, 180°, etc.), and their associated radian measures (\( \frac{\pi}{6} \), \( \frac{\pi}{4} \), \( \frac{\pi}{3} \), \( \frac{\pi}{2} \), \( \frac{2\pi}{3} \), \( \pi \), etc.) Show students where 1 radian fits. [L2] Visual-Spatial

Critical Thinking

“Natural” Angles To begin, ask students what they think is a “natural” way to measure angles. Next, on the board draw two radii in a large circle that are about 60° (\( \frac{\pi}{3} \) radians) apart in angle. Carefully obtain measurements for the radius and intercepted arc length. Ask students to compute the ratio of arc length to radius and to determine in which form the measure of the angle numerically is closest to this ratio. The ratio should be about 1, and \( \frac{\pi}{3} \) radians is then closest. On this basis, radians are the natural form of angle measure. [L2] Visual-Spatial

Angular Displacement

The Greek letter theta, \( \theta \), is used to represent the angle of revolution. Figure 8-1 shows the angles in radians for several common fractions of a revolution. Note that counterclockwise rotation is designated as positive, while clockwise is negative. As an object rotates, the change in the angle is called angular displacement.

As you know, Earth makes one complete revolution, or \( 2\pi \) rad, in 24 h. In 12 h, its rotation is through \( \pi \) rad. Through what angle does Earth rotate in 6 h? Because 6 h is one-fourth of a day, Earth rotates through an angle of \( \frac{\pi}{2} \) rad during that period. Earth’s rotation as seen from the north pole is positive. Is it positive or negative when viewed from the south pole?

How far does a point on a rotating object move? You already found that a point on the edge of an object moves \( \pi r \) when \( \theta \) rad. However, this is not the case. Radians indicate the ratio between \( d \) and \( r \). Thus, \( d \) is measured in m.

Angular Velocity

How fast does a CD spin? How do you determine its speed of rotation? Recall from Chapter 2 that velocity is displacement divided by the time taken to make the displacement. Likewise, the angular velocity of an object is angular displacement divided by the time taken to make the displacement. Thus, the angular velocity of an object is given by the following equation, where angular velocity is represented by the Greek letter omega, \( \omega \).

\[
\text{Angular Velocity of an Object } \omega = \frac{\Delta \theta}{\Delta t}
\]

The angular velocity is equal to the angular displacement divided by the time required to make the rotation.
Recall that if the velocity changes over a time interval, the average velocity is not equal to the instantaneous velocity at any given instant. Similarly, the angular velocity calculated in this way is actually the average angular velocity over a time interval, $\Delta t$. Instantaneous angular velocity is equal to the slope of a graph of angular position versus time.

Angular velocity is measured in rad/s. For Earth, $\omega_E = (2\pi \text{ rad})/ (24.0 \text{ h})(3600 \text{ s/h}) = 7.27 \times 10^{-5} \text{ rad/s}$. In the same way that counterclockwise rotation produces positive angular displacement, it also results in positive angular velocity.

If an object's angular velocity is $\omega$, then the linear velocity of a point a distance, $r$, from the axis of rotation is given by $v = r\omega$. The speed at which an object on Earth's equator moves as a result of Earth's rotation is given by $v = r\omega = (6.38 \times 10^6 \text{ m})/(7.27 \times 10^{-5} \text{ rad/s}) = 464 \text{ m/s}$. Earth is an example of a rotating, rigid body. Even though different points on Earth rotate different distances in each revolution, all points rotate through the same angle. All parts of a rigid body rotate at the same rate. The Sun, on the other hand, is not a rigid body. Different parts of the Sun rotate at different rates. Most objects that we will consider in this chapter are rigid bodies.

### Angular Acceleration

What if angular velocity is changing? For example, if a car were accelerated from 0.0 m/s to 25 m/s in 15 s, then the angular velocity of the wheels also would change from 0.0 rad/s to 78 rad/s in the same 15 s. The wheels would undergo angular acceleration, which is defined as the change in angular velocity divided by the time required to make the change. Angular acceleration, $\alpha$, is represented by the following equation.

**Angular Acceleration of an Object**

$$\alpha = \frac{\Delta \omega}{\Delta t}$$

Angular acceleration is equal to the change in angular velocity divided by the time required to make that change.

Angular acceleration is measured in rad/s$^2$. If the change in angular velocity is positive, then the angular acceleration also is positive. Angular acceleration defined in this way is also the average angular acceleration over the time interval $\Delta t$. One way to find the instantaneous angular acceleration is to find the slope of a graph of angular velocity as a function of time. The linear acceleration of a point at a distance, $r$, from the axis of an object with angular acceleration, $\alpha$, is given by $a = r\alpha$. Table 8-1 is a summary of linear and angular relationships.

#### Table 8-1

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Linear $d$ (m)</th>
<th>Angular $\theta$ (rad)</th>
<th>Relationship $d = r\theta$</th>
<th>$v = r\omega$</th>
<th>$a = r\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>$d$ (m)</td>
<td>$\theta$ (rad)</td>
<td>$d = r\theta$</td>
<td>$v = r\omega$</td>
<td>$a = r\alpha$</td>
</tr>
<tr>
<td>Velocity</td>
<td>$v$ (m/s)</td>
<td>$\omega$ (rad/s)</td>
<td>$v = r\omega$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Acceleration</td>
<td>$a$ (m/s$^2$)</td>
<td>$\alpha$ (rad/s$^2$)</td>
<td>$a = r\alpha$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Angular Displacement**

**Estimated Time** 10 minutes

**Materials** bicycle wheel, meterstick, and cloth measuring tape

**Procedure** Measure the radius of the wheel. Mark one point on the side wall of the wheel at the outer edge. Place the wheel on the ground such that the marked point is at the bottom. Roll the wheel along the ground by one rotation and measure the distance along the ground that the wheel traveled. Use this information to relate $\theta$ to distance moved. The distance traveled will be $2\pi r$. 

---

**Identifying Misconceptions**

**Average and Instantaneous** As in the case of linear motion, average and instantaneous values of an object’s angular velocity and angular acceleration are often confused. If the angular velocity is changing, one can always define an average angular velocity, but for each instant, one can also define an instantaneous angular velocity. If the angular position is plotted as a function of time, then the instantaneous angular velocity would be the tangent to the graph. If the angular velocity is changing at a constant rate, then the average and instantaneous angular acceleration will be the same. 

**L2 Logical-Mathematical**

- **Using Table 8-1**

Create a drawing on the chalkboard illustrating the connection between angular and linear displacement, and showing how $d$ increases with $r$ for fixed $\theta$. Illustrate, using motion diagrams, the similar relation between $v$ and $r$ for fixed $\omega$.

**L2 Visual-Spatial**

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Page 91, FAST FILE

Chapters 6–10 Resources
1. What is the angular displacement of each of the following hands of a clock in 1 h? 
State your answer in three significant digits.
   a. the second hand
   b. the minute hand
   c. the hour hand
2. If a truck has a linear acceleration of 1.85 m/s² and the wheels have an angular acceleration of 5.23 rad/s², what is the diameter of the truck’s wheels?
3. The truck in the previous problem is towing a trailer with wheels that have a diameter of 48 cm.
   a. How does the linear acceleration of the trailer compare with that of the truck?
   b. How do the angular accelerations of the wheels of the trailer and the wheels of the truck compare?
4. You want to replace the tires on your car with tires that have a larger diameter. After you change the tires, for trips at the same speed and over the same distance, how will the angular velocity and number of revolutions change?

Angular frequency A rotating object can make many revolutions in a given amount of time. For instance, a spinning wheel can go through several complete revolutions in 1 min. Thus, the number of complete revolutions made by the object in 1 s is called angular frequency. Angular frequency is \( f = \frac{\omega}{2\pi} \). In the next section, you will explore the factors that cause the angular frequency to change.

3 ASSESS
Check for Understanding
Demonstrating Angular Velocity Use a large wheel, preferably one with a smaller pulley attached. Mark a spot on the outer edge. Wrap a string around the wheel. Pull the end of the string at a constant velocity. Ask students to observe the angular velocity of the wheel. Repeat pulling the string at about the same constant velocity as before. This time have it wrapped around the smaller pulley. But first, ask them to predict if the angular velocity will change. Angular velocity: \( \omega = \frac{v}{r} \). Because the radius is smaller, angular velocity must be larger.

Extension
Determining Angular Acceleration Ask students to suppose that a massive object is hung on a string attached to a wheel. How can probeware be used to measure the angular acceleration of the wheel? A rotational motion sensor can be connected to the apparatus. Ask whether the linear acceleration will be different if a pulley is used. \( \alpha = \frac{\Delta \omega}{\Delta t} \). The relationship depends on \( r \). When the pulley is used, \( a \) will be much smaller.
8.2 Rotational Dynamics

How do you start the rotation of an object? That is, how do you change its angular velocity? Suppose you have a soup can that you want to spin. If you wrap a string around it and pull hard, you could make the can spin rapidly. Later in this chapter, you will learn why gravity, the force of Earth’s mass on the can, acts on the center of the can. The force of the string, on the other hand, is exerted at the outer edge of the can, and at right angles to the line from the center of the can, to the point where the string leaves the can’s surface.

You have learned that a force changes the velocity of a point object. In the case of a soup can, a force that is exerted in a very specific way changes the angular velocity of an extended object, which is an object that has a definite shape and size. Consider how you open a door: you exert a force. How can you exert the force to open the door most easily? To get the most effect from the least force, you exert the force as far from the axis of rotation as possible, as shown in Figure 8-3. In this case, the axis of rotation is an imaginary vertical line through the hinges. The doorknob is near the outer edge of the door. You exert the force on the doorknob at right angles to the door, away from the hinges. Thus, the magnitude of the force, the distance from the axis to the point where the force is exerted, and the direction of the force determine the change in angular velocity.

**Lever arm** For a given applied force, the change in angular velocity depends on the lever arm, which is the perpendicular distance from the axis of rotation to the point where the force is exerted. If the force is perpendicular to the radius of rotation, as it was with the soup can, then the lever arm is the distance from the axis, \( r \). For the door, it is the distance from the hinges to the point where you exert the force, as illustrated in Figure 8-4a, on the next page. If the force is not perpendicular, the perpendicular component of the force must be found.

The force exerted by the string around the can is perpendicular to the radius. If a force is not exerted perpendicular to the radius, however, the lever arm is reduced. To find the lever arm, extend the line of the force until it forms a right angle with a line from the center of rotation. The distance between the intersection and the axis is the lever arm. Thus, using trigonometry, the lever arm, \( L \), can be calculated by the equation \( L = r \sin \theta \), as shown in Figure 8-4b. In this equation, \( r \) is the distance from the axis of rotation to the point where the force is exerted, and \( \theta \) is the angle between the force and the radius from the axis of rotation to the point where the force is applied.

**Objectives**
- Describe torque and the factors that determine it.
- Calculate net torque.
- Calculate the moment of inertia.

**Vocabulary**
- lever arm
- torque
- moment of inertia
- Newton’s second law for rotational motion

![Figure 8-3](image-url) When opening a door that is free to rotate about its hinges, the greatest torque is produced when the force is applied farthest from the hinges (a), at an angle perpendicular to the door (b).

**Section 8.2 Rotational Dynamics**

<table>
<thead>
<tr>
<th>No effect</th>
<th>Little effect</th>
<th>Maximum effect</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image-url" alt="Door" /></td>
<td><img src="image-url" alt="Door" /></td>
<td><img src="image-url" alt="Door" /></td>
</tr>
</tbody>
</table>

**1 FOCUS**

**Bellringer Activity**

**Lever Arm** Use a classroom door or other massive object that pivots about one edge. Have students explore how the location and direction of the force they exert changes the rotation of the door. Show how the greatest rotation for the least force occurs when the force is exerted perpendicular to the door, as far from the hinge as possible.

**Tie to Prior Knowledge**

**Force** Students will use the concept of angular motion, Newton’s second law, and the geometry of the circle to develop Newton’s second law for rotational motion.

**Technology**
- TeacherWorks™ CD-ROM
- Interactive Chalkboard CD-ROM
- ExamView® Pro Testmaker CD-ROM
- physicspp.com
- physicspp.com/vocabulary_puzzlemaker

**2 TEACH**

**Concept Development**

**Maximum Torque** When a string is wrapped around an object, it is always tangent to the circle and thus perpendicular to the radius. The torque exerted when you exert a force \( F \) on the string is \( \tau = rF \).
Explore Figure 8-4b

Explore the equation for torque: \( \tau = Fr \sin \theta \). Note that it can be written \( \tau = r(F \sin \theta) \) or \( \tau = F(r \sin \theta) \). In the first case, it can be interpreted as a force, reduced because of the angle at which it is exerted, times a distance. In the second, it is a force times a lever arm, which depends both on the distance and the angle at which the force is exerted.

**Logical-Mathematical**

### Discussion

**Question** At what point between the hinged side and outside edge of a door would a given force perpendicular to the door produce the same torque as a force of equal strength but at a 30° angle between the force and the radius of rotation is not equal to 90° (b).

**Answer** The torques would be equal if the force were applied halfway across the door.

---

**In-Class Example**

**Question** What torque is applied when the same wrench (as in the example problem) is used with a 35-N·m torque applied at an angle 75° from the perpendicular?

**Answer** Draw a sketch. If the angle is 75° from the perpendicular, then it is 15° from the radius arm. Therefore, the lever arm, \( r \sin \theta = (0.25 \text{ m})(0.26) = 0.065 \text{ m} \). The force needed is then \( \frac{35 \text{ N} \cdot \text{m}}{0.065 \text{ m}} = 5.4 \times 10^2 \text{ N} \).

---

### Example Problem 1

**Lever Arm** A bolt on a car engine needs to be tightened with a torque of 35 N·m. You use a 25-cm-long wrench and pull on the end of the wrench at an angle of 60.0° from the perpendicular. How long is the lever arm, and how much force do you have to exert?

1. **Analyze and Sketch the Problem**
   - Sketch the situation. Find the lever arm by extending the force vector backwards until a line that is perpendicular to it intersects the axis of rotation.
   - **Known:**
     - \( r = 0.25 \text{ m} \)
     - \( \tau = 35 \text{ N} \cdot \text{m} \)
     - \( \theta = 60.0° \)
   - **Unknown:**
     - \( L = ? \)
     - \( F = ? \)

2. **Solve for the Unknown**
   - Solve for the length of the lever arm.
     
     \[
     L = r \sin \theta \\
     = (0.25 \text{ m})(\sin 60.0°) \\
     = 0.22 \text{ m}
     \]
   - Solve for the force.
     
     \[
     \tau = Fr \sin \theta \\
     F = \frac{\tau}{r \sin \theta} \\
     = \frac{35 \text{ N} \cdot \text{m}}{(0.25 \text{ m})(\sin 60.0°)} \\
     = 1.6 \times 10^2 \text{ N}
     \]

3. **Evaluate the Answer**
   - **Are the units correct?** Force is measured in newtons.
   - **Does the sign make sense?** Only the magnitude of the force needed to rotate the wrench clockwise is calculated.

---

**Math Handbook**

Trigonometric Ratios

---

**Visually Impaired** Feel the torque! Pivot a broomstick around one end. Screw hooks into the bottom at a variety of distances from the pivot. Let a student with visual impairment test-lift a heavy object. Hang the object from a selected hook while the student lifts the free end of the broomstick. Ask the student to describe the exerted force on the broomstick. Describe to the student the location of the hook. Repeat, using a different hook, and ask the student to relate the torque and the lever arm. Alternatively, have a student pull down on a hook, using a scale to exert a fixed force but adjusting the angle to vary the torque.
11. Consider the wrench in Example Problem 1. What force is needed if it is applied to the wrench at a point perpendicular to the wrench?

12. If a torque of 55.0 N·m is required and the largest force that can be exerted by you is 135 N, what is the length of the lever arm that must be used?

13. You have a 0.234-m-long wrench. A job requires a torque of 32.4 N·m, and you can exert a force of 232 N. What is the smallest angle, with respect to the vertical, at which the force can be exerted?

14. You stand on the pedal of a bicycle. If you have a mass of 65 kg, the pedal makes an angle of 35° above the horizontal, and the pedal is 18 cm from the center of the chain ring, how much torque would you exert?

15. If the pedal in problem 14 is horizontal, how much torque would you exert? How much torque would you exert when the pedal is vertical?

Finding Net Torque

Try the following experiment. Get two pencils, some coins, and some transparent tape. Tape two identical coins to the ends of the pencil and balance it on the second pencil, as shown in Figure 8-5. Each coin exerts a torque that is equal to its weight, $F_g$, times the distance, $r$, from the balance point to the center of the coin, as follows:

$$\tau = F_g r$$

But the torques are equal and opposite in direction. Thus, the net torque is zero:

$$\tau_1 - \tau_2 = 0$$

or

$$F_{g1} r_1 - F_{g2} r_2 = 0$$

How can you make the pencil rotate? You could add a second coin on top of one of the two coins, thereby making the two forces different. You also could slide the balance point toward one end or the other of the pencil, thereby making the two distances different.

**Figure 8-5** The torque exerted by the first coin, $F_{g1} r_1$, is equal and opposite in direction to the torque exerted by the second coin, $F_{g2} r_2$ when the pencil is balanced.

---

**The Torque Wrench** Applying the correct torque when tightening a bolt is very important. Too little torque and the bolt will not exert enough force to keep the two objects together. On the other hand, if you apply too much torque you can break the bolt. A torque wrench is designed to keep a person from applying too much torque. The simplest models have a flexible handle and a thin indicator that converts the amount that the handle has flexed into the torque. The more expensive models have a device that produces an audible click when the correct torque has been applied. Many electric and pneumatic wrenches have built-in torque indicators to keep excessive torque from being applied.
Discussion

Question Tell students to imagine a very heavy, uniform plank of length \( l \) and mass \( m_1 \) placed so that it extends a distance, \( r \), over the edge of a small platform. The plank is not fastened to the platform in any way. Ask students what factors determine the maximum value of \( r \) so that a person of mass \( m_2 \) can walk out to the end of the plank. Have students develop an equation for this maximum distance.

Answer The length of the plank, the positioning of the center of the plank with respect to the edge of the platform, and the mass of the person determine the maximum distance. Develop the equation by first recognizing that the plank is uniform, so the mass can be considered to act at its center. The balance point is the edge of the platform. Then assume that the person is at the end of the plank while it is positioned for maximum distance, \( r \), just before the plank tips over. Net torque is zero. Setting clockwise torque equal to counterclockwise torque, then solving for \( r \), yields

\[
mg \left( \frac{l}{2} - r \right) = m_2gr.
\]

\[
r = \frac{m_1}{m_1 + m_2} \frac{l}{2}\]

Seesaw All changes in rotational motion are the result of net torque. When considering the case of the seesaw, you may need to remind some students that torques—vector–like forces—in rotational dynamics—add, so they can be balanced or unbalanced. If the two torques do not balance, then the board rotates. Two people on opposite ends of a seesaw exert torques in opposite directions. If the torques they exert are equal and opposite, the board does not rotate. If the two people have equal weights, then the board balances when they sit equal distances from the axis—or fulcrum, in this case. If they are not of equal weights, then the heavier person must sit closer to the axis in order to balance the board.

Chapter 8 Rotational Motion

EXAMPLE Problem 2

Balancing Torques Kariann (56 kg) and Aysha (43 kg) want to balance on a 1.75-m-long seesaw. Where should they place the pivot point?

1. Analyze and Sketch the Problem
   - Sketch the situation.
   - Draw and label the vectors.

   Known: Unknown:
   \[
m_k = 56 \text{ kg} \quad r_k = ?
\]
   \[
m_A = 43 \text{ kg} \quad r_A = ?
\]
   \[
r_k + r_A = 1.75 \text{ m}
\]

2. Solve for the Unknown
   Find the two forces.
   Kariann:
   \[
   F_{gK} = m_kg = (56 \text{ kg})(9.80 \text{ m/s}^2) = 5.5 \times 10^2 \text{ N}
   \]
   Aysha:
   \[
   F_{gA} = m_Ag = (43 \text{ kg})(9.80 \text{ m/s}^2) = 4.2 \times 10^2 \text{ N}
   \]
   Define Kariann’s distance in terms of the length of the seesaw and Aysha’s distance.
   \[
r_k = 1.75 \text{ m} - r_A
\]
   When there is no rotation, the sum of the torques is zero.
   \[
   F_{gK}r_k - F_{gA}r_A = 0.0 \text{ N·m}
   \]
   Solve for \( r_k \).
   \[
   F_{gK}(1.75 \text{ m}) - F_{gA}(r_A) = 0.0 \text{ N·m}
   \]
   \[
   F_{gA} = \frac{F_{gK}(1.75 \text{ m})}{(1.75 \text{ m}) - r_A}
   \]
   \[
   = \frac{(5.5 \times 10^2 \text{ N})(1.75 \text{ m})}{(5.5 \times 10^2 \text{ N} + 4.2 \times 10^2 \text{ N})}
   \]
   \[
   = 0.99 \text{ m}
   \]

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16. Ashok, whose mass is 43 kg, sits 1.8 m from the center of a seesaw. Steve, whose mass is 52 kg, wants to balance Ashok. How far from the center of the seesaw should Steve sit?

17. A bicycle-chain wheel has a radius of 7.7 cm. If the chain exerts a 35.0-N force on the wheel in the clockwise direction, what torque is needed to keep the wheel from turning?

18. Two baskets of fruit hang from strings going around pulleys of different diameters, as shown in Figure 8-6. What is the mass of basket A?

19. Suppose the radius of the larger pulley in problem 18 was increased to 6.0 cm. What is the mass of basket A now?

20. A bicyclist, of mass 65.0 kg, stands on the pedal of a bicycle. The crank, which is 0.170 m long, makes a 45.0° angle with the vertical, as shown in Figure 8-7. The crank is attached to the chain wheel, which has a radius of 9.70 cm. What force must the chain exert to keep the wheel from turning?

The Moment of Inertia

If you exert a force on a point mass, its acceleration will be inversely proportional to its mass. How does an extended object rotate when a torque is exerted on it? To observe firsthand, recover the pencil, the coins, and the transparent tape that you used earlier in this chapter. First, tape the coins at the ends of the pencil. Hold the pencil between your thumb and forefinger, and wiggle it back and forth. Take note of the forces that your thumb and forefinger exert. These forces create torques that change the angular velocity of the pencil and coins.

Now move the coins so that they are only 1 or 2 cm apart. Wiggle the pencil as before. Did the amount of torque and force need to be changed? The torque that was required was much less this time. Thus, the amount of mass is not the only factor that determines how much torque is needed to change angular velocity; the location of that mass also is relevant.

The resistance to rotation is called the moment of inertia, which is represented by the symbol $I$ and has units of mass times the square of the distance. For a point object located at a distance, $r$, from the axis of rotation, the moment of inertia is given by the following equation.

**Moment of Inertia of a Point Mass**  

$$I = mr^2$$

The moment of inertia of a point mass is equal to the mass of the object times the square of the object’s distance from the axis of rotation.
**Concept Development**

**Integration of Mass** Calculation of the moment of inertia requires the use of integral calculus. Conceptually, the object is broken into tiny blocks of mass. The tiny mass is multiplied by the square of its distance from the axis. Those products are summed over all the tiny mass blocks.

**Reinforcement**

**Demonstrate** If two objects have the same mass but different shapes, the one with its mass distributed farther from its axis of rotation will have a larger moment of inertia. Show students pairs of objects of the same mass but different shape, such as a ring or a disk. Have them pick the one that should have the larger moment of inertia. They should be able to say that the ring shape has a larger moment of inertia.

**Kinesthetic**

**Page 57, FAST FILE**

**Chapters 6–10 Resources**

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**Table 8-2**

<table>
<thead>
<tr>
<th>Object</th>
<th>Location of Axis</th>
<th>Diagram</th>
<th>Moment of Inertia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thin hoop of radius ( r )</td>
<td>Through central diameter</td>
<td><img src="image" alt="Diagram" /></td>
<td>( mr^2 )</td>
</tr>
<tr>
<td>Solid, uniform cylinder of radius ( r )</td>
<td>Through center</td>
<td><img src="image" alt="Diagram" /></td>
<td>( \frac{1}{2}mr^2 )</td>
</tr>
<tr>
<td>Uniform sphere of radius ( r )</td>
<td>Through center</td>
<td><img src="image" alt="Diagram" /></td>
<td>( \frac{2}{5}mr^2 )</td>
</tr>
<tr>
<td>Long, uniform rod of length ( l )</td>
<td>Through center</td>
<td><img src="image" alt="Diagram" /></td>
<td>( \frac{1}{12}ml^2 )</td>
</tr>
<tr>
<td>Long, uniform rod of length ( l )</td>
<td>Through end</td>
<td><img src="image" alt="Diagram" /></td>
<td>( \frac{1}{3}ml^2 )</td>
</tr>
<tr>
<td>Thin, rectangular plate of length ( l ) and width ( w )</td>
<td>Through center</td>
<td><img src="image" alt="Diagram" /></td>
<td>( \frac{1}{12}ml(f^2 + w^2) )</td>
</tr>
</tbody>
</table>

---

As you have seen, the moment of inertia for complex objects, such as the pencil and coins, depends on how far the coins are from the axis of rotation. A bicycle wheel, for example, has almost all of its mass in the rim and tire. Its moment of inertia is almost exactly equal to \( mr^2 \), where \( r \) is the radius of the wheel. For most objects, however, the mass is distributed continuously and so the moment of inertia is less than \( mr^2 \). For example, as shown in Table 8-2, for a solid cylinder of radius \( r \), \( I = \frac{1}{2}mr^2 \), while for a solid sphere, \( I = \frac{2}{5}mr^2 \).

The moment of inertia also depends on the location of the rotational axis, as illustrated in Figure 8-8. To observe this firsthand, hold a book in the upright position, by placing your hands at the bottom of the book. Feel the torque needed to rock the book toward you, and then away from you. Now put your hands in the middle of the book and feel the torque needed to rock the book toward you and then away from you. Note that much less torque is needed when your hands are placed in the middle of the book because the average distance of the book’s mass from the rotational axis is much less in this case.

---

**Hands-On with Moment of Inertia** Have students investigate how the moment of inertia affects the velocity of an object. Have students use a hoop and a disk that are of the same diameter, width, and mass. Ask students which object has its mass located farthest from its center. The hoop What you are really asking students is to identify how each object’s mass is distributed around its axis of rotation. Ask students to predict which object will reach the bottom of an incline first. Have them test their predictions. Instruct students to place both the disk and the hoop on an incline and to release them simultaneously. The solid disk will reach the bottom first.

**Linguistic**

---

**Figure 8-8** The moment of inertia of a book depends on the axis of rotation. The moment of inertia of the book in (a) is larger than the moment of inertia of the book in (b) because the average distance of the book’s mass from the rotational axis is larger.

---

**Diagram for Figure 8-8**

- **Diagram (a)**: A book with its mass distributed near the center of the page. The moment of inertia is smaller.
- **Diagram (b)**: A book with its mass distributed farthest from the center of the page. The moment of inertia is larger.
**EXAMPLE Problem 3**

**Moment of Inertia** A simplified model of a twirling baton is a thin rod with two round objects at each end. The length of the baton is 0.65 m, and the mass of each object is 0.30 kg. Find the moment of inertia of the baton when it is rotated about the midpoint between the round objects. What is the moment of inertia of the baton when it is rotated around one end? Which is greater? Neglect the mass of the rod.

**1 Analyze and Sketch the Problem**
- Sketch the situation. Show the baton with the two different axes of rotation and the distances from the axes of rotation to the masses.

**Known:**
- $m = 0.30$ kg
- $I$ = 0.65 m

**Unknown:** $I = ?$

**2 Solve for the Unknown**
Calculate the moment of inertia of each mass separately.

Rotating about the center of the rod:

$$r = \frac{1}{2}l$$
$$= \frac{1}{2}(0.65 \text{ m})$$
$$= 0.33 \text{ m}$$

$$I_{\text{single mass}} = mr^2$$
$$= (0.30 \text{ kg})(0.33 \text{ m})^2$$
$$= 0.033 \text{ kg m}^2$$

Find the moment of inertia of the baton.

$$I = 2I_{\text{single mass}}$$
$$= 2(0.033 \text{ kg m}^2)$$
$$= 0.066 \text{ kg m}^2$$

Rotating about one end of the rod:

$$I_{\text{single mass}} = mr^2$$
$$= (0.30 \text{ kg})(0.65 \text{ m})^2$$
$$= 0.13 \text{ kg m}^2$$

Find the moment of inertia of the baton.

$$I = I_{\text{single mass}}$$
$$= 0.13 \text{ kg m}^2$$

The moment of inertia is greater when the baton is swung around one end.

**3 Evaluate the Answer**
- *Are the units correct?* Moment of inertia is measured in kg m$^2$.
- *Is the magnitude realistic?* Masses and distances are small, and so are the moments of inertia. Doubling the distance increases the moment of inertia by a factor of 4. Thus, doubling the distance overcomes having only one mass contributing.

---

**Question** If the baton in Example Problem 3 is rotated about a point one-fourth of the distance from one round object, how does the moment of inertia compare to the two examples?

**Answer** One round object is one-fourth the length of the rod from the axis; the other is three-fourths the length. Therefore,

$$I = m\left(\frac{l}{4}\right)^2 + m\left(\frac{3}{4}l\right)^2 = m\left(\frac{13}{16}l\right)^2$$

So, $I = (0.30 \text{ kg})\left(\frac{13}{16}\right)(0.65 \text{ m})^2 = 0.10 \text{ kg m}^2$, intermediate between the two other cases.

---

**Using Models**

**Distributed Mass Model**
Duct tape, quarters, a meterstick, and a stopwatch can be used to model the effect of changing moment of inertia on resistance to rotation. First, tape quarters at the 40-cm and 60-cm marks on a meterstick. **CAUTION: Wear goggles.** Have one student hold the meterstick horizontally at the 50-cm mark with an extended arm. Have another student use a stopwatch to record the amount of time required for the first student to oscillate the meterstick back and forth, from horizontal to vertical, five times, as quickly as possible. Then repeat with quarters set at the 25-cm and 75-cm marks, and again at the 10-cm and 90-cm marks. Ask students what happens to the period of oscillation (time required per oscillation) as the mass is distributed farther from the center of rotation. The period should increase quickly, with the second increment larger than the first, provided the student uses a similar twisting effort in all three trials. This illustrates that as point masses are distributed farther from the axis of rotation, their resistance to rotation increases faster than a linear relationship would suggest.
21. When \( r \) is doubled, \( I \) is multiplied by a factor of 4.

22. The moment of inertia is larger when the mass that is located far from the center.

23. The moments of inertia are different. If the spacing between spheres is \( r \) and each sphere has mass \( m \), then rotation about sphere \( A \) is \( I = m r^2 + m(2r)^2 = 5m r^2 \). Rotation about sphere \( C \) is \( I = m r^2 + m r^2 = 2m r^2 \). The moment of inertia is greater when rotating around sphere \( A \).

24. About sphere \( A \): 0.020 kg-m\(^2\); About sphere \( C \): 0.0080 kg-m\(^2\).

### Newton’s Second Law for Rotational Motion

Newton’s second law for linear motion is expressed as \( a = F/m \). If you rewrite this equation to represent rotational motion, acceleration is replaced by angular acceleration, \( \alpha \), force is replaced by net torque, \( \tau \), and mass is replaced by moment of inertia, \( I \). Thus, **Newton’s second law for rotational motion** states that angular acceleration is directly proportional to the net torque and inversely proportional to the moment of inertia. This law is expressed by the following equation.

\[
\alpha = \frac{\tau}{I}
\]

Recall the coins taped on the pencil. To change the direction of rotation of the pencil—to give it angular acceleration—you had to apply torque to the pencil. The greater the moment of inertia, the more torque needed to produce the same angular acceleration.

### Practice Problems

#### Challenge Problem

Rank the objects shown in the diagram according to their moments of inertia about the indicated axes. All spheres have equal masses and all separations are the same.

\[
\begin{align*} 
A & > B \\
C & > D \\
& > \text{A} > \text{B} > \text{C} > \text{D}
\end{align*}
\]
Torque  A solid steel wheel has a mass of 15 kg and a diameter of 0.44 m. It starts at rest. You want to make it rotate at 8.0 rev/s in 15 s.

a. What torque must be applied to the wheel?
b. If you apply the torque by wrapping a strap around the outside of the wheel, how much force should you exert on the strap?

1 Analyze and Sketch the Problem

- Sketch the situation. The torque must be applied in a counterclockwise direction; force must be exerted as shown.

   Known:  
   - $m = 15$ kg
   - $r = \frac{1}{2}(0.44$ m$) = 0.22$ m
   - $\omega_i = 0.0$ rad/s
   - $\omega_f = 2\pi$(8.0 rev/s)
   - $t = 15$ s

   Unknown:  
   - $\alpha = ?$
   - $I = ?$
   - $\tau = ?$
   - $F = ?$

2 Solve for the Unknown

a. Solve for angular acceleration.

$$\alpha = \frac{\Delta \omega}{\Delta t}$$

Substitute $\omega_f = 2\pi$(8.0 rev/s), $\omega_i = 0.0$ rad/s

$$\alpha = \frac{2\pi(8.0 \text{ rev/s}) - (0.0 \text{ rad/s})}{15 \text{ s}} = 3.4 \text{ rad/s}^2$$

Solve for the moment of inertia.

$$I = \frac{1}{2}mr^2$$

Substitute $m = 15$ kg, $r = 0.22$ m

$$I = \frac{1}{2}(15 \text{ kg})(0.22 \text{ m})^2 = 0.36 \text{ kg\cdot m}^2$$

Solve for torque.

$$\tau = I\alpha$$

Substitute $I = 0.36 \text{ kg\cdot m}^2$, $\alpha = 3.4 \text{ rad/s}^2$

$$\tau = 0.36 \text{ kg\cdot m}^2 \times 3.4 \text{ rad/s}^2 = 1.2 \text{ kg\cdot m}^2/\text{s}^2 = 1.2 \text{ N\cdot m}$$

b. Solve for force.

$$F = \frac{\tau}{r}$$

Substitute $\tau = 1.2 \text{ N\cdot m}$, $r = 0.22$ m

$$F = \frac{1.2 \text{ N\cdot m}}{0.22 \text{ m}} = 5.5 \text{ N}$$

3 Evaluate the Answer

- **Are the units correct?** Torque is measured in N·m and force is measured in N.
- **Is the magnitude realistic?** Despite its large mass, the small size of the wheel makes it relatively easy to spin.

---

**Challenge Activity**

Can Competition  Have students race two soup cans, a liquid broth soup versus a thick paste soup, down an inclined plank. Ask students which one wins. Why? Have students run the race with cans of different radii, analyze the results, and present them to the class. The key to understanding is to pick the axis of rotation where the can touches the incline. Sum the moment of inertia of a point object at the center of the can to the moment of inertia of the can, so, $I = mr^2 + I_{\text{can}}$. Then, Newton’s second law leads to $a = g\sin\theta/(1 + I_{\text{can}}/mr^2)$. The liquid broth soup can acts like a hoop because the liquid does not rotate in the short race time. The paste soup can acts as a solid cylinder, so it wins. The radius of the can does not affect acceleration.

---

**In-Class Example**

Identifying Misconceptions

Inertia and Torque  Moment of inertia for a point mass has two factors, the value of the mass and the square of its distance from the axis of rotation. Because torque involves force times just the first power of distance, confusion may result. The following hint may help students sort this out. First, remind students that angular acceleration is related to linear acceleration, $a = r\alpha$. Next, students should apply Newton’s second law for linear motion, $F = ma$. Now have them multiply both sides of this equation by $r$, $Fr = mr^2\alpha$. So, $\alpha = \frac{Fr}{mr^2} = \frac{\tau}{I}$. Angular acceleration, torque, and moment of inertia, as defined, make Newton’s second law work for rotational motion.
25. Consider the wheel in Example Problem 4. If the force on the strap were twice as great, what would be the speed of rotation of the wheel after 15 s?

26. A solid wheel accelerates at 3.25 rad/s² when a force of 4.5 N exerts a torque on it. If the wheel is replaced by a wheel with all of its mass on the rim, the moment of inertia is given by \( I = mr^2 \). If the same angular velocity were desired, what force would have to be exerted on the strap?

27. A bicycle wheel can be accelerated either by pulling on the chain that is on the gear or by pulling on a string wrapped around the tire. The wheel’s radius is 0.38 m, while the radius of the gear is 0.14 m. If you obtained the needed acceleration with a force of 15 N on the chain, what force would you need to exert on the string?

28. The bicycle wheel in problem 27 is used with a smaller gear whose radius is 0.11 m. The wheel can be accelerated either by pulling on the chain that is on the gear or by pulling string that is wrapped around the tire. If you obtained the needed acceleration with a force of 15 N on the chain, what force would you need to exert on the string?

29. A disk with a moment of inertia of 0.26 kg m² is attached to a smaller disk mounted on the same axle. The smaller disk has a diameter of 0.180 m and a mass of 2.5 kg. A strap is wrapped around the smaller disk, as shown in Figure 8-10. Find the force needed to give this system an angular acceleration of 2.57 rad/s².

In summary, changes in the amount of torque applied to an object, or changes in the moment of inertia, affect the rate of rotation. In this section, you learned how Newton’s second law of motion applies to rotational motion. In the next section, you will learn how to keep objects from rotating.

30. Torque Vijesh enters a revolving door that is not moving. Explain where and how Vijesh should push to produce a torque with the least amount of force.

31. Lever Arm You try to open a door, but you are unable to push at a right angle to the door. So, you push the door at an angle of 55° from the perpendicular. How much harder would you have to push to open the door just as fast as if you were to push it at 90°?

32. Net Torque Two people are pulling on ropes wrapped around the edge of a large wheel. The wheel has a mass of 12 kg and a diameter of 2.4 m. One person pulls in a clockwise direction with a 43-N force, while the other pulls in a counterclockwise direction with a 67-N force. What is the net torque on the wheel?

33. Moment of Inertia Refer to Table 8-2 on page 206 and rank the moments of inertia from least to greatest of the following objects: a sphere, a wheel with almost all of its mass at the rim, and a solid disk. All have equal masses and diameters. Explain the advantage of using the one with the least moment of inertia.

34. Newton’s Second Law for Rotational Motion A rope is wrapped around a pulley and pulled with a force of 13.0 N. The pulley’s radius is 0.150 m. The pulley’s rotational speed goes from 0.0 to 14.0 rev/min in 4.50 s. What is the moment of inertia of the pulley?

35. Critical Thinking A ball on an extremely low-friction, tilted surface, will slide downhill without rotating. If the surface is rough, however, the ball will roll. Explain why, using a free-body diagram.
Why are some vehicles more likely than others to roll over when involved in an accident? What causes a vehicle to roll over? The answer lies in the design of the vehicle. In this section, you will learn some of the factors that cause an object to tip over.

The Center of Mass

How does an object rotate around its center of mass? A wrench may spin about its handle or end-over-end. Does any single point on the wrench follow a straight path? Figure 8-1a shows the path of the wrench. You can see that there is a single point whose path traces a straight line, as if the wrench could be replaced by a point particle at that location. The center of mass of an object is the point on the object that moves in the same way that a point particle would move.

Locating the center of mass

How can you locate the center of mass of an object? First, suspend the object from any point. When the object stops swinging, the center of mass is along the vertical line drawn from the suspension point as shown in Figure 8-1b. Draw the line. Then, suspend the object from another point. Again, the center of mass must be below this point. Draw a second vertical line. The center of mass is at the point where the two lines cross, as shown in Figure 8-1c. The wrench, racket, and all other freely rotating objects rotate about an axis that goes through their center of mass. Where is the center of mass of a person located?
Center of Mass

**Estimated Time**: 10 minutes

**Materials**: large piece of foam (10 cm thick, about 30 × 50 cm), two chemiluminescent glow sticks (approximately 15 cm long)

**Procedure**

1. Activate the glow sticks. Punch a hole through the foam near an edge and push a glow stick through. Find the center of mass of the foam plus glow stick. Mark the center, punch a hole through, and press the second glow stick through the hole. If the glow sticks are insufficiently heavy, add weights such as lead sinkers.
2. Toss the foam across the room, making it spin. Ask what path it took. Did all parts follow the same path? an arc; no
3. Then turn off the room lights. Toss the foam again. Ask students to describe the path. The glow stick at the center of mass followed a parabolic path; the other executed circles around it.

**Tipping** Obtain boxes of different shapes. Fill some with moderately heavy and stable materials that can’t move inside the box. Have students try tipping the boxes over (carefully) and analyze when they are stable and when they are not. Challenge them to find the center of mass of the boxes. Kinesthetic

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**Center of Mass and Stability**

What factors determine whether a vehicle is stable or prone to roll over in an accident? To understand the problem, think about tipping over a box. A tall, narrow box, standing on end, tips more easily than a low, broad box. Why? To tip a box, as shown in Figure 8-13, you must rotate it about a corner. You pull at the top with a force, \( F \), applying a torque, \( \tau_F \). The weight of the box, \( F_g \), applies an opposing torque, \( \tau_w \). When the center of mass is directly above the point of support, \( \tau_w \) is zero. The only torque is the one applied by you. As the box rotates farther, its center of mass is no longer above its base of support, and both torques act in the same direction. At this point, the box tips over rapidly.

**Figure 8-13** The bent arrows show the direction of the torque produced by the force exerted to tip over a box.

**Center of Mass and Center of Gravity** For a solid object, one can replace the entire mass of the object with a point mass equal to that of the object’s mass at the center of mass. The object will rotate about its center of mass. The center of mass also is the balance point of the object. That is, the object could be suspended from its center of mass and it would not rotate. It would not be out of balance. Likewise, all lines of suspension of a mass must pass through the center of mass because there can be no net torque on the object when it is hanging in equilibrium. This is the basis of the method used for finding center of mass through intersecting lines of suspension. The terms “center of mass” and “center of gravity” are used interchangeably.
**Stability** An object is said to be stable if an external force is required to tip it. The box in Figure 8-13 is stable as long as the direction of the torque due to its weight, \( \tau_w \), tends to keep it upright. This occurs as long as the box’s center of mass lies above its base. To tip the box over, you must rotate its center of mass around the axis of rotation until it is no longer above the base of the box. To rotate the box, you must lift its center of mass. The broader the base, the more stable the object is. For this reason, if you are standing on a bus that is weaving through traffic and you want to avoid falling down, you need to stand with your feet spread apart.

Why do vehicles roll over? Figure 8-14 shows two vehicles rolling over. Note that the one with the higher center of mass does not have to be tilted very far for its center of mass to be outside its base—its center of mass does not have to be raised as much as the other vehicle’s. The lower the location of an object’s center of mass, the greater its stability.

You are stable when you stand flat on your feet. When you stand on tiptoe, however, your center of mass moves forward directly above the balls of your feet, and you have very little stability. A small person can use torque, rather than force, to defend himself or herself against a stronger person. In judo, aikido, and other martial arts, the fighter uses torque to rotate the opponent into an unstable position, where the opponent’s center of mass does not lie above his or her feet.

In summary, if the center of mass is outside the base of an object, it is unstable and will roll over without additional torque. If the center of mass is above the base of the object, it is stable. If the base of the object is very narrow and the center of mass is high, then the object is stable, but the slightest force will cause it to tip over.

### Conditions for Equilibrium

If your pen is at rest, what is needed to keep it at rest? You could either hold it up or place it on a desk or some other surface. An upward force must be exerted on the pen to balance the downward force of gravity. You must also hold the pen so that it will not rotate. An object is said to be in static equilibrium if both its velocity and angular velocity are zero or constant. Thus, for an object to be in static equilibrium, it must meet two conditions. First, it must be in translational equilibrium; that is, the net force exerted on the object must be zero. Second, it must be in rotational equilibrium; that is, the net torque exerted on the object must be zero.

![Figure 8-14](image-url) Larger vehicles have a higher center of mass than smaller ones. The higher the center of mass, the smaller the tilt needed to cause the vehicle’s center of mass to move outside its base and cause the vehicle to roll over.

**Spinning Tops**

1. Cut out two cardboard disks of 10-cm and 15-cm diameter.
2. Use a pencil with an eraser that has rounded edges from use. If it is new, rub it on paper to round it.
3. Spin the pencil and try to make it stand on the eraser. Repeat several times and record your observations.
4. Carefully push the pencil through the center of the 10-cm disk.
5. Spin the pencil with the disk and try to make it stand on the eraser.
6. Move the disk to different points on the pencil. Spin and record your observations.
7. Repeat steps 4-6 with the 15-cm disk.

**Analyze and Conclude**

8. Sequence the three trials in order from least to most stable.
9. Describe the location of the pencil’s center of mass.
10. Analyze the placement of the disk and its effect on stability.

**Expected Results** The pencil will fall over and not stand on end. The larger the disk and the lower the disk is on the pencil, the more stable it will be when rotating and the longer (in general) it will spin.

**Analyze and Conclude**

8. Pencil without disks, pencil with 10-cm disk, pencil with 15-cm disk
9. The pencil’s center of mass is in the middle or center of the pencil.
10. Answers may vary. The cardboard increases the mass and, when placed near the table, lowers the object’s center of mass. It is harder to tip over due to its inertia from spinning.

**Balance and the Center of Mass**

**Estimated Time** 5 minutes

**Materials** none

**Procedure** Have a student stand with his or her toes against a wall and attempt to stand on tiptoe. He or she will find it extremely difficult, if not impossible. Discuss why this is.
**Static Equilibrium** A 5.8-kg ladder, 1.80 m long, rests on two sawhorses. Sawhorse A is 0.60 m from one end of the ladder, and sawhorse B is 0.15 m from the other end of the ladder. What force does each sawhorse exert on the ladder?

**1 Analyze and Sketch the Problem**
- Sketch the situation.
- Choose the axis of rotation at the point where $F_A$ acts on the ladder. Thus, the torque due to $F_A$ is zero.

<table>
<thead>
<tr>
<th>Known:</th>
<th>Unknown:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m = 5.8 \text{ kg}$</td>
<td>$F_A = ?$</td>
</tr>
<tr>
<td>$l = 1.80 \text{ m}$</td>
<td>$F_B = ?$</td>
</tr>
<tr>
<td>$l_A = 0.60 \text{ m}$</td>
<td></td>
</tr>
<tr>
<td>$l_B = 0.15 \text{ m}$</td>
<td></td>
</tr>
</tbody>
</table>

**2 Solve for the Unknown**
For a ladder that has a constant density, the center of mass is at the center rung.

The net force is the sum of all forces on the ladder.

$$F_{\text{net}} = F_A + F_B + (-F_g)$$

The ladder is in translational equilibrium, so the net force exerted on it is zero.

Solve for $F_A$.

$$F_A = F_B - F_g$$

Find the torques due to $F_g$ and $F_B$.

$$\tau_g = -r_B F_g \quad \text{\tau}_g \text{ is in the clockwise direction.}$$

$$\tau_B = +r_B F_B \quad \text{\tau}_B \text{ is in the counterclockwise direction.}$$

The net torque is the sum of all torques on the object.

$$\tau_{\text{net}} = \tau_B + \tau_g$$

The ladder is in rotational equilibrium, so $\tau_{\text{net}} = 0.0 \text{ N\cdot m}$.

Solve for $F_B$.

$$F_B = \frac{\tau_B}{r_B}$$

Using the expression $F_A = F_B - F_g$, substitute in the expressions for $F_B$ and $F_g$.

$$F_A = F_B - F_g$$

$$= F_B - \frac{r_B \cdot mg}{r_B} \quad \text{Substitute } F_B = \frac{r_B \cdot mg}{r_B}$$

$$= mg - \frac{r_B \cdot mg}{r_B} \quad \text{Substitute } F_B = mg$$

$$= mg \left(1 - \frac{r_B}{r_B}\right)$$

**Critical Thinking**

**Static Equilibrium** Tell students that static means “unchanging state.” Then ask if this definition also means that there must be no forces acting on an object in static equilibrium. **No!** Emphasize that there are forces acting on objects in static equilibrium and that these forces are balanced so that their net force is zero. The word equilibrium comes from two Latin words meaning “even balance.”

**Concept Development**

**Center of Mass** Ask students where they would find the center of mass for a ladder. **the center rung**, assuming of course a constant density. Ask students to draw a free-body diagram of the forces on a ladder resting on two sawhorses. Designate forces: the left sawhorse is $F_A$ and the right sawhorse is $F_B$. These forces are parallel to each other and act in the upward direction. Ask students to write out the net force acting on the ladder for the ladder to be in equilibrium. $F_A + F_B - F_g = 0$, or $F_A + F_B = F_g$ **Visual-Spatial**

**Architects and Engineers** Statics can be used to analyze forces acting on objects in equilibrium. Architects and civil engineers use statics to determine the structural integrity of their designs. Using accurate calculations is essential; failure to calculate can be a disastrous embarrassment, if not a fatal and negligent error. One large public university was building a new campus. The library was to be covered in brick, as were all the buildings on campus in keeping with the city’s tradition. Unfortunately, the engineers and architects neglected to add the weight of the books in the library stacks. As a result, the weight of the books caused the floors to bow which in turn caused the outside walls to buckle—the result was popping bricks!
36. What would be the forces exerted by the two sawhorses if the ladder in Example Problem 5 had a mass of 11.4 kg?

37. A 73-kg ladder, 1.92 m long, rests on two sawhorses, as shown in Figure 8-15.

Sawhorse A, on the left, is located 0.30 m from the end, and sawhorse B, on the right, is located 0.45 m from the other end. Choose the axis of rotation to be the center of mass of the ladder.

a. What are the forces acting on the ladder?

b. Write the equation for rotational equilibrium.

c. Solve the equation for $F_A$ in terms of $F_B$.

d. How would the forces exerted by the two sawhorses change if A were moved very close to, but not directly under, the center of mass?

38. A 45-m-long wooden plank with a 24-kg mass is supported in two places.

One support is directly under the center of the board, and the other is at one end. What are the forces exerted by the two supports?

39. A 85-kg diver walks to the end of a diving board. The board, which is 3.5 m long with a mass of 14 kg, is supported at the center of mass of the board and at one end. What are the forces on the two supports?

**Concept Development**

**Torque** Recall for students that torque is defined as the product of the force, $F$, and the lever arm, $r (\tau = Fr)$. In the Example Problem on page 214, $F_A$, $F_B$, and $F_g$ are each perpendicular to the ladder. Ask students to define the lever arm of each force. It is the distance along the ladder from the axis of rotation to the point where each force is acting.

**Critical Thinking**

**Rotational Equilibrium** Refer students again to the In-Class Example on p. 214. Point out that the equation $F_A + F_B - F_g = 0$ (or, $F_A + F_B = F_g$) shows the net force acting on the ladder. Ask them how to find $F_A$ and $F_B$. Use the second condition of static equilibrium: The ladder must be in rotational equilibrium. Ask students what the requirement is for the ladder to be in rotational equilibrium. The net torque on it must be zero.

**Simultaneous Equations** Equilibrium problems usually involve an equation for the net force—a sum that must be zero for equilibrium—and another for the net torque, which likewise must be zero. Most of the problems students confront involve objects with uniform mass distribution, a separate weight that acts at some particular point along the object, and two points of support. They should understand that choosing the axis of rotation at a point where one of the forces acts, or at the center of mass of the distributed object, will simplify the problem. Make sure that students can use these principles in order to set up the correct equations. Then, address difficulties they may have completing the algebraic solution.
Newton’s laws hold only in fixed reference frames. Nonetheless important because public misconception, precisely because it feels so real. An understanding of apparent forces is nonetheless important because Newton’s laws hold only in fixed reference frames. Reinforcement

Apparent Forces Centrifugal and Coriolis “forces” seem to arise in rotating reference frames, but in each case, the effect is actually subjective from the standpoint of the person or object within the reference frame. The centrifugal “force,” for example, is a major public misconception, precisely because it feels so real. An understanding of apparent forces is nonetheless important because Newton’s laws hold only in fixed reference frames.

Rotating Frames of Reference

When you are on a one a rapidly spinning amusement-park ride, it feels like a strong force is pushing you to the outside. A pebble on the floor of the ride would accelerate outward without a horizontal force being exerted on it in the same direction. The pebble would not move in a straight line. In other words, Newton’s laws would not apply. This is because rotating frames of reference are accelerated frames. Newton’s laws are valid only in inertial or nonaccelerated frames.

Motion in a rotating reference frame is important to us because Earth rotates. The effects of the rotation of Earth are too small to be noticed in the classroom or lab, but they are significant influences on the motion of the atmosphere and therefore, on climate and weather.

Centrifugal “Force”

Suppose you fasten one end of a spring to the center of a rotating platform. An object lies on the platform and is attached to the other end of the spring. As the platform rotates, an observer on the platform sees the object stretch the spring. The observer might think that some force toward the outside of the platform is pulling on the object. This apparent force is called centrifugal “force.” It is not a real force because there is no physical outward push on the object. Still, this “force” seems real, as anyone who has ever been on an amusement-park ride can attest.

As the platform rotates, an observer on the ground sees things differently. This observer sees the object moving in a circle. The object accelerates toward the center because of the force of the spring. As you know, the acceleration is centripetal acceleration and is given by $a_c = v^2/r$. It also can be written in terms of angular velocity, as $a_c = \omega^2/r$. Centripetal acceleration is proportional to the distance from the axis of rotation and depends on the square of the angular velocity. Thus, if you double the rotational frequency, the acceleration increases by a factor of 4.

The Coriolis “Force”

A second effect of rotation is shown in Figure 8-16. Suppose a person standing at the center of a rotating disk throws a ball toward the edge of the disk. Consider the horizontal motion of the ball as seen by two observers and ignore the vertical motion of the ball as it falls.

![Figure 8-16](image)

**Coriolis “Force”**

**Purpose** to observe the Coriolis “force”

**Materials** rotating stool; 2 × 10 lumber; 2.5 m long; heavy clamps to attach lumber to stool; ball

**CAUTION:** Ensure that the apparatus is stable and secure enough to withstand the stress of the activity. Emphasize to students that the rotation not need be very fast.

**Procedure** Have a student sit on each end of the lumber and gently toss the ball back and forth. Slowly rotate them. Have them toss the ball again. What do they observe? The ball will appear to the students to be traveling a curved path. Students standing on the ground will see that it goes in a straight path.

**Assessment** Have students draw a top view of the lab.
40. Center of Mass Can the center of mass of an object be located in an area where the object has no mass? Explain.

41. Stability of an Object Why is a modified vehicle with its body raised high on risers less stable than a similar vehicle with its body at normal height?

42. Conditions for Equilibrium Give an example of an object for each of the following conditions.
   a. rotational equilibrium, but not translational equilibrium
   b. translational equilibrium, but not rotational equilibrium

43. Center of Mass Where is the center of mass of a roll of masking tape?

44. Locating the Center of Mass Describe how you would find the center of mass of this textbook.

45. Rotating Frames of Reference A penny is placed on a rotating, old-fashioned record turntable. At the highest speed, the penny starts sliding outward. What are the forces acting on the penny?

46. Critical Thinking You have learned why the winds around a low-pressure area move in a counterclockwise direction. Would the winds move in the same or opposite direction in the southern hemisphere? Explain.

3 ASSESS

Check for Understanding

Stability Review stability and equilibrium by asking students how they stand to keep from falling over. Have a student kneel on the floor. Place the elbows against the knees and forearms along the floor. Have someone place a small box at the person’s fingertips. Then, moving the hands behind the back, try to touch the box with the nose. Generally, a male is unstable because he must move his higher center of mass in front of his knees. Females can usually do this trick.

Reteach

Equilibrium Demo Ask students what is needed to keep at rest an extended object that is at rest. When force components in all three directions sum to zero, the object will not accelerate. Demonstrate the disruption of equilibrium by balancing a meterstick on your finger, then rolling the finger slightly so that the stick rotates. Discuss torques and what forces are at work when the meterstick is in equilibrium.
Translational and Rotational Equilibrium

For maintenance on large buildings, scaffolding can be hung on the outside. In order for the scaffolding to support workers, it must be in translational and rotational equilibrium. If two or more forces act on the scaffolding, each can produce a rotation about either end. Scaffolding with uniform mass distribution acts as though all of the mass is concentrated at its center. In translational equilibrium the object is not accelerating; thus, the upward and downward forces are equal. In order to achieve rotational equilibrium, the sum of all the clockwise torques must equal the sum of all the counterclockwise torques as measured from a pivot point. That is, the net torque must be zero. In this lab you will model scaffolding hung from two ropes using a meterstick and spring scales, and use numbers to measure the forces on the scaffolding.

QUESTION

What conditions are required for equilibrium when parallel forces act on an object?

Objectives

- Collect and organize data about the forces acting on the scaffolding.
- Describe clockwise and counterclockwise torque.
- Compare and contrast translational and rotational equilibrium.

Safety Precautions

- Use care to avoid dropping masses.

Materials

- meterstick
- two Buret clamps
- two 0-5 N spring scales
- 500-g hooked mass
- two ring stands
- 200-g hooked mass

The left spring scale will be considered a pivot point for the purposes of this lab. Therefore, the lever arm will be measured from this point.

1. Place the ring stands 80 cm apart.
2. Attach a Buret clamp to each of the ring stands.
3. Verify that the scales are set to zero before use. If the scales need to be adjusted, ask your teacher for assistance.
4. Hang a spring scale from each Buret clamp attached to a ring stand.
5. Hook the meterstick onto the spring scale in such a manner that the 10-cm mark is supported by one hook and the 90-cm mark is supported by the other hook.
6. Read each spring scale and record the force in Data Table 1.
7. Hang a 500-g mass on the meterstick at the 30-cm mark. This point should be 20 cm from the left scale.
8. Read each spring scale and record the force in Data Table 1.
9. Hang a 200-g mass on the meterstick at the 70-cm mark. This point should be 60 cm from the left scale.
10. Read each spring scale and record the force in Data Table 1.

Sample Data

Data Table 1

<table>
<thead>
<tr>
<th>Object Added</th>
<th>Distance From Left Scale (m)</th>
<th>Left Scale Reading (N)</th>
<th>Right Scale Reading (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meterstick</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>500-g mass</td>
<td>0.20</td>
<td>4.1</td>
<td>1.7</td>
</tr>
<tr>
<td>200-g mass</td>
<td>0.60</td>
<td>4.6</td>
<td>3.1</td>
</tr>
</tbody>
</table>

Data Table 2

<table>
<thead>
<tr>
<th>Object Added</th>
<th>( \tau_C ) (N\cdot m)</th>
<th>( \tau_{CC} ) (N\cdot m)</th>
<th>Lever Arm (m)</th>
<th>Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meterstick</td>
<td>x</td>
<td>0.40</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>500-g mass</td>
<td>x</td>
<td>0.20</td>
<td>4.9</td>
<td></td>
</tr>
<tr>
<td>200-g mass</td>
<td>x</td>
<td>0.60</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>Right Scale</td>
<td>x</td>
<td>0.80</td>
<td>3.1</td>
<td></td>
</tr>
</tbody>
</table>

Data Table 3

<table>
<thead>
<tr>
<th>Object Added</th>
<th>( \tau_C ) (N\cdot m)</th>
<th>( \tau_{CC} ) (N\cdot m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meterstick</td>
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<td></td>
</tr>
<tr>
<td>500-g mass</td>
<td>0.98</td>
<td></td>
</tr>
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<td>200-g mass</td>
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<tr>
<td>Right Scale</td>
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<td></td>
</tr>
<tr>
<td>( \Sigma \tau )</td>
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<td>2.5</td>
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</table>
Data Table 1

<table>
<thead>
<tr>
<th>Object Added</th>
<th>Distance From Left Scale (m)</th>
<th>Left Scale Reading (N)</th>
<th>Right Scale Reading (N)</th>
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</thead>
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<td></td>
</tr>
<tr>
<td>500-g mass</td>
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<td></td>
</tr>
<tr>
<td>200-g mass</td>
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<td>0.6</td>
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</tbody>
</table>

Data Table 2

<table>
<thead>
<tr>
<th>Object Added</th>
<th>( \tau_c ) (N m)</th>
<th>( \tau_{cc} ) (N m)</th>
<th>Lever Arm (m)</th>
<th>Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meterstick</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>500-g mass</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>200-g mass</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Right scale</td>
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</tbody>
</table>

Data Table 3

<table>
<thead>
<tr>
<th>Object Added</th>
<th>( \sum \tau ) (N m)</th>
<th>( \sum \tau_{cc} ) (N m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meterstick</td>
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<td></td>
</tr>
<tr>
<td>500-g mass</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200-g mass</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Right scale</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Analyze

1. **Calculate** Find the mass of the meterstick.
2. **Calculate** Find the force, or weight, that results from each object and record it in Data Table 2. For the right scale, read the force it exerts and record it in Data Table 2.
3. Using the point where the left scale is attached as a pivot point, identify the forces located elsewhere that cause the scaffold to rotate clockwise or counterclockwise. Mark these in Data Table 2 with an \( x \).
4. Record the lever arm distance of each force from the pivot point in Data Table 2.
5. **Use Numbers** Calculate the torque for each object by multiplying the force and lever arm distance. Record these values in Data Table 3.

Compare and contrast the sum of the clockwise torques, \( \sum \tau_c \), and the counterclockwise torques, \( \sum \tau_{cc} \).

3. **Calculate** Find the mass of the meterstick.
4. **Calculate** Find the force, or weight, that results from each object and record it in Data Table 2. For the right scale, read the force it exerts and record it in Data Table 2.
5. **Use Numbers** Calculate the torque for each object by multiplying the force and lever arm distance. Record these values in Data Table 3.

3. Compare and contrast the sum of the clockwise torques, \( \sum \tau_c \), and the counterclockwise torques, \( \sum \tau_{cc} \).
4. What is the percent difference between \( \sum \tau_c \) and \( \sum \tau_{cc} \)?

Conclude and Apply

1. Is the system in translational equilibrium? How do you know?
2. Draw a free-body diagram of your system, showing all the forces.

Real-World Physics

Research the safety requirements in your area for putting up, using, and dismantling scaffolding.

Real-World Physics

Answers will vary. Answers should include information relating directly to the lab. For example:

- Stationary scaffolds over 38.1 m in height and rolling scaffolds over 18.3 m in height must be designed by a professional engineer. Ties, guys, bracing and/or outriggers may be needed to assure a safe, stable scaffold assembly.
- The height of the scaffold in relation to the minimum base width, wind loads, the use of brackets or cantilevered platforms, and imposed scaffold loads determine the need for stability bracing.
- The bottom tie must be placed no higher than four times the minimum base width and every 7.9 m vertically thereafter. Ties should be placed as close to the top of the scaffold as possible and, in no case, less than four times (three times in California) the minimum base width of the scaffold from the top.

Going Further

Use additional masses at locations of your choice with your teacher’s permission and record your data.

Answers will vary depending on masses selected and their locations.

To Make this Lab an Inquiry Lab: As a class or in groups, have students brainstorm questions about equilibrium they want to explore. Allow students to choose the question they want to explore and to work with their groups to design a procedure to test it. Be sure to review their plans for safety considerations.
Background
This article is based on the most recent available information provided by the NHTSA (National Highway Transportation Safety Administration), a division of the U.S. Department of Transportation. Rollovers accounted for more than 10,000 fatalities in the United States in 1999. Rollovers happen in highly unusual circumstances. They occur when vehicles are taken to their limits and when very excessive speeds are coupled with very sharp turning maneuvers. Does riding higher off the ground present a higher risk for rollovers? The answer depends on how the vehicle performs, road conditions, safety precautions taken, driver awareness and training programs, etc. It is true that a vehicle with a higher center of gravity has a greater propensity to roll over when compared to a vehicle with a lower center of gravity given the same speed and conditions.

Teaching Strategies
The question of sport-utility vehicle safety is a complex issue that warrants further research. You may wish to have students write freely for five minutes on the topic: What I think engineers can do to improve the safety of sport-utility vehicles. Have students do another free-write on the same topic after they complete research.

Discussion
Discuss ways to reduce rollover risk in a sport-utility vehicle or other automobile. For example:
1) Always wear a safety belt. NHTSA estimates that vehicle occupants who are wearing safety belts are far more likely to survive a rollover accident than unbelted riders.
2) Avoid conditions that lead to loss of control, such as driving under the influence, driving too fast, and driving with worn or improperly inflated tires.
3) Load vehicles properly. Consult the owner’s manual regarding maximum load and load distribution. Be careful not to overload a roof rack, since any load on the roof raises the center of gravity.
4) Be careful on rural roads to avoid running off the road and striking a ditch, rut, or embankment.
5) Avoid panic-like steering since it can lead to loss of control.

Why are sport-utility vehicles more flippable? Many believe that the large size of the sport-utility vehicle makes it more stable and secure. But, a sport-utility vehicle, as well as other tall vehicles such as vans, is much more likely to roll over than a car.

The Problem
A sport-utility vehicle has a high center of mass which makes it more likely to topple. Another factor that affects rollover is the static stability factor, which is the ratio of the track width to the center of mass. Track width is defined as half the distance between the two front wheels. The higher the static stability factor, the more likely a vehicle will stay upright.

Many sport-utility vehicles have a center of mass 13 or 15 cm higher than passenger cars. Their track width, however, is about the same as that of passenger cars. Suppose the stability factor for a sport-utility vehicle is 1.06 and 1.43 for a car. Statistics show that in a single-vehicle crash, the sport-utility vehicle has a 37 percent chance of rolling over, while the car has a 10.6 percent chance of rolling over.

However, the static stability factor oversimplifies the issue. Weather and driver behavior are also contributors to rollover crashes. Vehicle factors, such as tires, suspension systems, inertial properties, and advanced handling systems all play a role as well.

It is true that most rollover crashes occur when a vehicle swerves off the road and hits a rut, soft soil, or other surface irregularity. This usually occurs when a driver is not paying proper attention or is speeding. Safe drivers greatly reduce their chances of being involved in a rollover accident by paying attention and driving at the correct speed. Still, weather and driver behavior being equal, the laws of physics indicate that sport-utility vehicles carry an increased risk.

What Is Being Done? Some models are being built with wider track widths or stronger roofs. Optional side-curtain air bags have sensors to keep the bags inflated for up to 6 s, rather than the usual fraction of a second. This will cushion passengers if the vehicle should flip several times.

The Stability of Sport-Utility Vehicles
An ESC system processes information from the sensors and automatically applies the brakes to individual wheels when instability is detected.

A promising new technology called Electronic Stability Control (ESC) can be used to prevent rollover accidents. An ESC system has electronic sensors that detect when a vehicle begins to spin due to oversteering, and also when it begins to slide in a plowlike manner because of understeering. In these instances, an ESC system automatically applies the brakes at one or more wheels, thereby reorienting the vehicle in the right direction.

Safe driving is the key to preventing many automobile accidents. Knowledge of the physics behind rollover accidents and the factors that affect rollover accidents may help make you an informed, safe driver.

Going Further
1. Hypothesize In a single-vehicle accident, sport-utility vehicles generally fare better than the passenger cars involved in the accident. Why is this so?
2. Debate the Issue ESC is a life-saving technology. Should it be mandatory in all sport-utility vehicles? Why or why not?

Going Further
1. Sport-utility vehicles are heavier than most passenger cars; they are also higher, so passengers tend to be located above the areas of greatest impact.
2. Accept all reasonable answers.
### 8.1 Describing Rotational Motion

**Vocabulary**
- radian (p. 197)
- angular displacement (p. 198)
- angular velocity (p. 198)
- angular acceleration (p. 199)

**Key Concepts**
- Angular position and its changes are measured in radians. One complete revolution is $2\pi$ rad.
- Angular velocity is given by the following equation.
  \[ \omega = \frac{\Delta \theta}{\Delta t} \]
- Angular acceleration is given by the following equation.
  \[ \alpha = \frac{\Delta \omega}{\Delta t} \]
- For a rotating, rigid object, the angular displacement, velocity, and acceleration can be related to the linear displacement, velocity, and acceleration for any point on the object.
  \[ \Delta s = r \theta \quad v = r \omega \quad a = r \alpha \]

### 8.2 Rotational Dynamics

**Vocabulary**
- lever arm (p. 201)
- torque (p. 202)
- moment of inertia (p. 205)
- Newton’s second law for rotational motion (p. 208)

**Key Concepts**
- When torque is exerted on an object, its angular velocity changes.
- Torque depends on the magnitude of the force, the distance from the axis of rotation at which it is applied, and the angle between the force and the radius from the axis of rotation to the point where the force is applied.
  \[ \tau = Fr \sin \theta \]
- The moment of inertia of an object depends on the way the object’s mass is distributed about the rotational axis. For a point object:
  \[ I = mr^2 \]
- Newton’s second law for rotational motion states that angular acceleration is directly proportional to the net torque and inversely proportional to the moment of inertia.
  \[ \alpha = \frac{\tau_{net}}{I} \]

### 8.3 Equilibrium

**Vocabulary**
- center of mass (p. 211)
- centrifugal “force” (p. 216)
- Coriolis “force” (p. 217)

**Key Concepts**
- The center of mass of an object is the point on the object that moves in the same way that a point particle would move.
- An object is stable against rollover if its center of mass is above its base.
- An object is in equilibrium if there are no net forces exerted on it and if there are no net torques acting on it.
- Centrifugal “force” and the Coriolis “force” are two apparent forces that appear when a rotating object is analyzed from a coordinate system that rotates with it.
Mastering Concepts

48. A bicycle wheel rotates at a constant 25 rev/min. Is its angular velocity decreasing, increasing, or constant? (8.1)

49. A toy rotates at a constant 5 rev/min. Is its angular acceleration positive, negative, or zero? (8.1)

50. Do all parts of Earth rotate at the same rate? Explain. (8.1)

51. A unicycle wheel rotates at a constant 14 rev/min. Is the total acceleration of a point on the tire inward, outward, tangential, or zero? (8.1)

52. Think about some possible rotations of your textbook. Are the moments of inertia about these three axes the same or different? Explain. (8.2)

53. Torque is important when tightening bolts. Why is force not important? (8.2)

54. Rank the torques on the five doors shown in Figure 8-18 from least to greatest. Note that the magnitude of all the forces is the same. (8.2)

55. Explain how you can change an object’s angular frequency. (8.2)

56. To balance a car’s wheel, it is placed on a vertical shaft and weights are added to make the wheel horizontal. Why is this equivalent to moving the center of mass until it is at the center of the wheel? (8.3)

57. A stunt driver maneuvers a monster truck so that it is traveling on only two wheels. Where is the center of mass of the truck? (8.3)

58. Suppose you stand flat-footed, then you rise and balance on tiptoe. If you stand with your toes touching a wall, you cannot balance on tiptoe. Explain. (8.3)

59. Why does a gymnast appear to be floating on air when she raises her arms above her head in a leap? (8.3)

60. Why is a vehicle with wheels that have a large diameter more likely to roll over than a vehicle with wheels that have a smaller diameter? (8.3)

Applying Concepts

61. Two gears are in contact and rotating. One is larger than the other, as shown in Figure 8-19. Compare their angular velocities. Also compare the linear velocities of two teeth that are in contact.

62. Videotape When a videotape is rewound, why does it wind up fastest towards the end?

63. Spin Cycle What does a spin cycle of a washing machine do? Explain in terms of the forces on the clothes and water.

64. How can you experimentally find the moment of inertia of an object?

65. Bicycle Wheels Three bicycle wheels have masses that are distributed in three different ways: mostly at the rim, uniformly, and mostly at the hub. The wheels all have the same mass. If equal torques are applied to them, which one will have the greatest angular acceleration? Which one will have the least?

66. Bowling Ball When a bowling ball leaves a bowler’s hand, it does not spin. After it has gone about half the length of the lane, however, it does spin. Explain how its rotation rate increased and why it does not continue to increase.

69. She moves her center of mass closer to her head.

70. The center of mass of the vehicle with the larger wheels is located at a higher point. Thus it does not have to be tilted very far before it rolls over.

Applying Concepts

61. The teeth have identical linear velocities. Because the radii are different and \( \omega = \frac{v}{r} \), the angular velocities are different.

62. The machine turns the spool at a constant angular velocity. Towards the end, the spool has the greatest radius. Because \( v = r\omega \), the velocity of the tape is fastest when the radius is greatest.
67. **Flat Tire** Suppose your car has a flat tire. You get out your tools and find a lug wrench to remove the nuts off the bolt studs. You find it impossible to turn the nuts. Your friend suggests ways you might produce enough torque to turn them. What three ways might your friend suggest?

68. **Tightrope Walkers** Tightrope walkers often carry long poles that sag so that the ends are lower than the center as shown in Figure 8-20. How does such a pole increase the tightrope walker’s stability? Hint: Consider both center of mass and moment of inertia.

69. **Merry-Go-Round** While riding a merry-go-round, you toss a key to a friend standing on the ground. For your friend to be able to catch the key, should you toss it a second or two before you reach the spot where your friend is standing or wait until your friend is directly behind you? Explain.

70. Why can you ignore forces that act on the axis of rotation of an object in static equilibrium when determining the net torque?

71. In solving problems about static equilibrium, why is the axis of rotation often placed at a point where one or more forces are acting on the object?

### Mastering Problems

#### 8.1 Describing Rotational Motion

72. A wheel is rotated so that a point on the edge moves through 1.50 m. The radius of the wheel is 2.50 m, as shown in Figure 8-21. Through what angle (in radians) is the wheel rotated?

73. The outer edge of a truck tire that has a radius of 45 cm has a velocity of 23 m/s. What is the angular velocity of the tire in rad/s?

74. A steering wheel is rotated through 128°, as shown in Figure 8-22. Its radius is 22 cm. How far would a point on the steering wheel’s edge move?

75. **Propeller** A propeller spins at 1880 rev/min.
   a. What is its angular velocity in rad/s?
   b. What is the angular displacement of the propeller in 2.50 s?

76. The propeller in the previous problem slows from 475 rev/min to 187 rev/min in 4.00 s. What is its angular acceleration?

77. An automobile wheel with a 9.00 cm radius, as shown in Figure 8-23, rotates at 2.50 rad/s. How fast does a point 7.00 cm from the center travel?

78. **Washing Machine** A washing machine’s two spin cycles are 328 rev/min and 542 rev/min. The diameter of the drum is 0.43 m. What is the ratio of the centripetal accelerations for the fast and slow spin cycles? Recall that $a_c = \frac{v^2}{r}$ and $v = rw$.
   a. What is the ratio of the centripetal accelerations for the fast and slow spin cycles?
   b. What is the ratio of the linear velocity of an object at the surface of the drum for the fast and slow spin cycles?

79. Find the maximum centripetal acceleration in terms of $g$ for the washing machine in problem 78.

### Chapter 8 Assessment

63. In the spin cycle, the water and clothes undergo great centripetal accelerations. The drum can exert forces on the clothes, but when the water reaches the holes in the drum, no inward force can be exerted on it and it therefore moves in a straight line, out of the drum.

64. You can apply a known torque and measure the resulting angular acceleration.

65. The more mass there is far from the axis, the greater the moment of inertia. If torque is fixed, the greater the moment of inertia, the less the angular acceleration. Thus, the wheel with mass mostly at the hub has the least moment of inertia and the greatest angular acceleration. The wheel with mass mostly near the rim has the greatest moment of inertia and the least angular acceleration.

66. Its rotation rate can be increased only if a torque is applied to it. The frictional force of the alley on the ball provides this force. Once the ball is rolling so that there is no velocity difference between the surface of the ball and the alley, then there is no more frictional force and thus no more torque.

67. Put an extension pipe on the end of the wrench to increase the lever arm, exert your force at right angles to the wrench, or exert a greater force, perhaps by standing on the end of the wrench.

68. The pole increases moment of inertia because of its mass and length. The drooping ends of the pole bring the center of mass closer to the wire, thus reducing the torque on the walker. The increased moment of inertia and decreased torque both reduce the angular acceleration if the walker becomes unbalanced. The walker can also use the pole to easily shift the center of mass over the wire to compensate for instability.

69. You have forward tangential velocity, so the key will leave your hand with that velocity. Therefore, you should toss it early.

70. The torque caused by these forces is zero because the lever arm is zero.

71. That makes the torque caused by that force equal to zero, reducing the number of torques that must be calculated.
80. A laboratory ultracentrifuge is designed to produce a centripetal acceleration of $0.35 \times 10^6 \text{ g}$ at a distance of $2.50 \text{ cm}$ from the axis. What angular velocity in rev/min is required?

8.2 Rotational Dynamics

81. **Wrench** A bolt is to be tightened with a torque of $8.0 \text{ N} \cdot \text{m}$. If you have a wrench that is $0.35 \text{ m}$ long, what is the least amount of force you must exert?

82. What is the torque on a bolt produced by a $15-\text{ N}$ force exerted perpendicular to a wrench that is $25 \text{ cm}$ long, as shown in Figure 8-24?

83. A toy consisting of two balls, each $0.45 \text{ kg}$, at the ends of a $0.46-\text{ m}$-long, thin, lightweight rod is shown in Figure 8-25. Find the moment of inertia of the toy. The moment of inertia is to be found about the center of the rod.

84. A bicycle wheel with a radius of $38 \text{ cm}$ is given an angular acceleration of $2.67 \text{ rad/s}^2$ by applying a force of $0.35 \text{ N}$ on the edge of the wheel. What is the wheel’s moment of inertia?

85. **Toy Top** A toy top consists of a rod with a diameter of $8.0-\text{ mm}$ and a disk of mass $0.0125 \text{ kg}$ and a diameter of $3.5 \text{ cm}$. The moment of inertia of the rod can be neglected. The top is spun by wrapping a string around the rod and pulling it with a velocity that increases from zero to $3.0 \text{ m/s}$ over $0.50 \text{ s}$. What is the resulting angular velocity of the top? What force was exerted on the string?

86. A $12.5-\text{ kg}$ board, $4.00 \text{ m}$ long, is being held up on one end by Ahmed. He calls for help, and Judi responds. What is the least force that Judi could exert to lift the board to the horizontal position? What part of the board should she lift to exert this force? What is the greatest force that Judi could exert to lift the board to the horizontal position? What part of the board should she lift to exert this force?

87. Two people are holding up the ends of a $4.25-\text{ kg}$ wooden board that is $1.75 \text{ m}$ long. A $6.00-\text{ kg}$ box sits on the board, $0.50 \text{ m}$ from one end, as shown in Figure 8-26. What forces do the two people exert?

88. A car’s specifications state that its weight distribution is 53 percent on the front tires and 47 percent on the rear tires. The wheel base is $2.46 \text{ m}$. Where is the car’s center of mass?

### Mixed Review

89. A wooden door of mass, $m$, and length, $l$, is held horizontally by Dan and Ajit. Dan suddenly drops his end.

- a. What is the angular acceleration of the door just after Dan lets go?
- b. Is the acceleration constant? Explain.

90. **Topsoil** Ten bags of topsoil, each weighing $175 \text{ N}$, are placed on a $2.43-\text{ m}$-long sheet of wood. They are stacked $0.50 \text{ m}$ from one end of the sheet of wood, as shown in Figure 8-27. Two people lift the sheet of wood, one at each end. Ignoring the weight of the wood, how much force must each person exert?
91. **Basketball** A basketball is rolled down the court. A regulation basketball has a diameter of 24.1 cm, a mass of 0.60 kg, and a moment of inertia of $5.8 \times 10^{-3}$ kg m$^2$. The basketball’s initial velocity is 2.5 m/s.
   a. What is its initial angular velocity?
   b. The ball rolls a total of 12 m. How many revolutions does it make?
   c. What is its total angular displacement?

92. The basketball in the previous problem stops rolling after traveling 12 m.
   a. If its acceleration was constant, what was its angular acceleration?
   b. What torque was acting on it as it was slowing down?

93. A cylinder with a 50 m diameter, as shown in Figure 8-28, is at rest on a surface. A rope is wrapped around the cylinder and pulled. The cylinder rolls without slipping.
   a. After the rope has been pulled a distance of 2.50 m at a constant speed, how far has the center of mass of the cylinder moved?
   b. If the rope was pulled a distance of 2.50 m in 1.25 s, how fast was the center of mass of the cylinder moving?
   c. What is the angular velocity of the cylinder?

94. **Hard Drive** A hard drive on a modern computer spins at 7200 rpm (revolutions per minute). If the drive is designed to start from rest and reach operating speed in 1.5 s, what is the angular acceleration of the disk?

95. **Speedometers** Most speedometers in automobiles measure the angular velocity of the transmission and convert it to speed. How will increasing the diameter of the tires affect the reading of the speedometer?

96. A box is dragged across the floor using a rope that is a distance $h$ above the floor. The coefficient of friction is 0.35. The box is 0.50 m high and 0.25 m wide. Find the force that just tips the box.

97. The second hand on a watch is 12 mm long. What is the velocity of its tip?

98. **Lumber** You buy a 2.44-m-long piece of 10 cm $\times$ 10 cm lumber. Your friend buys a piece of the same size and cuts it into two lengths, each 1.22 m long, as shown in Figure 8-29. You each carry your lumber on your shoulders.
   a. Which load is easier to lift? Why?
   b. Both you and your friend apply a torque with your hands to keep the lumber from rotating. Which load is easier to keep from rotating? Why?

99. **Surfboard** Harris and Paul carry a surfboard that is 2.43 m long and weighs 143 N. Paul lifts one end with a force of 57 N.
   a. What force must Harris exert?
   b. What part of the board should Harris lift?

100. A steel beam that is 6.50 m long weighs 325 N. It rests on two supports, 3.00 m apart, with equal amounts of the beam extending from each end. Suki, who weighs 575 N, stands on the beam in the center and then walks toward one end. How close to the end can she come before the beam begins to tip?

**Thinking Critically**

101. **Apply Concepts** Consider a point on the edge of a rotating wheel.
   a. Under what conditions can the centripetal acceleration be zero?
   b. Under what conditions can the tangential (linear) acceleration be zero?
   c. Can the tangential acceleration be nonzero while the centripetal acceleration is zero? Explain.
   d. Can the centripetal acceleration be nonzero while the tangential acceleration is zero? Explain.

102. **Apply Concepts** When you apply the brakes in a car, the front end dips. Why?

**Thinking Critically**

103. If the rope was pulled a distance of 2.50 m in 0.001 s, how fast was the center of mass of the board moving?

104. a. 21 N-m
   b. The tension in the rope is 64 N.

**Level 2**

98. a. The masses are the same, so the weights are the same. Thus, the same upward force is required to lift each load.
   b. The longer piece of lumber would be easier to keep from rotating because it has a greater moment of inertia.

99. a. 86 N
   b. Harris has to lift 2.0 m from Paul’s end of the board.

100. Suki can move 0.848 m from the support, or $1.75 - 0.848 = 0.90$ m from the end.

101. **Apply Concepts** Consider a point on the edge of a rotating wheel.
   a. When $\omega = 0$
   b. When $\alpha = 0$
   c. When $\omega = 0$ instantaneously, but $\alpha$ is not zero, $\omega$ will keep changing.
   d. Yes, as long as $\omega$ is constant but not zero.

102. The road exerts a force on the tires that brings the car to rest. The center of mass is above the road. Therefore, there is a net torque on the car, causing it to rotate in the direction that forces the front down.

103. 420 N

104. a. 21 N-m
   b. The tension in the rope is 64 N.
103. Analyze and Conclude A banner is suspended from a horizontal, pivoted pole, as shown in Figure 8-30. The pole is 2.10 m long and weighs 175 N. The banner, which weighs 105 N, is suspended 1.80 m from the pivot point or axis of rotation. What is the tension in the cable supporting the pole?

104. Analyze and Conclude A pivoted lamp pole is shown in Figure 8-31. The pole weighs 27 N, and the lamp weighs 64 N.
   a. What is the torque caused by each force?
   b. Determine the tension in the rope supporting the lamp pole.

105. Analyze and Conclude Gerald and Evelyn carry the following objects up a flight of stairs: a large mirror, a dresser, and a television. Evelyn is at the front end, and Gerald is at the bottom end. Assume that both Evelyn and Gerald exert only upward forces.
   a. Draw a free-body diagram showing Gerald and Evelyn exerting the same force on the mirror.
   b. Draw a free-body diagram showing Gerald exerting more force on the bottom of the dresser.
   c. Where would the center of mass of the television have to be so that Gerald carries all the weight?

Writing in Physics
106. For a planet and a moon with identical densities, the Roche limit is 2.446 times the radius of the planet. Earth’s Roche limit is 18,470 km.

107. The force exerted by the ground on the tire accelerates the car. This force is produced by the engine. It creates the force by rotating the axle. The torque is equal to the force on the edge of the tire multiplied by the radius of the tire. Gears in the transmission may cause the force to change, but they do not change the torque. Therefore, the amount of torque created by the engine is delivered to the wheels.

Cumulative Review
108. a. 24 N
   b. 1.96 m/s²
109. 71.6°
110. 122 km/hr at 14.3 degrees west of north
111. 972 N

Use ExamView® Pro Testmaker CD-ROM to:
- Create multiple versions of tests.
- Create modified tests with one mouse click for struggling students.
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1. The illustration below shows two boxes on opposite ends of a board that is 3.0 m long. The board is supported in the middle by a fulcrum. The box on the left has a mass, \( m_1 \), of 25 kg, and the box on the right has a mass, \( m_2 \), of 15 kg. How far should the fulcrum be positioned from the left side of the board in order to balance the masses horizontally?

- 0.38 m
- 1.1 m
- 0.60 m
- 1.9 m

2. A force of 60 N is exerted on one end of a 1.0-m-long lever. The other end of the lever is attached to a rotating rod that is perpendicular to the lever. By pushing down on the end of the lever, you can rotate the rod. If the force on the lever is exerted at an angle of 30°, what torque is exerted on the lever? (\( \sin 30° = 0.5 \), \( \cos 30° = 0.87 \), \( \tan 30° = 0.58 \))

- 30 N
- 52 N
- 69 N

3. A child attempts to use a wrench to remove a nut on a bicycle. Removing the nut requires a torque of 10 N·m. The maximum force the child is capable of exerting at a 90° angle is 50 N. What is the length of the wrench the child must use to remove the nut?

- 0.1 m
- 0.2 m
- 0.15 m
- 0.25 m

4. A car moves a distance of 420 m. Each tire on the car has a diameter of 42 cm. Which of the following shows how many revolutions each tire makes as they move that distance?

- \( \frac{5.0 \times 10^3}{\pi} \) rev
- \( \frac{5.0 \times 10^2}{\pi} \) rev
- \( \frac{1.5 \times 10^2}{\pi} \) rev
- \( \frac{1.0 \times 10^3}{\pi} \) rev

5. A thin hoop with a mass of 5.0 kg rotates about a perpendicular axis through its center. A force of 25 N is exerted tangentially to the hoop. If the hoop’s radius is 2.0 m, what is its angular acceleration?

- 1.3 rad/s
- 5.0 rad/s
- 2.5 rad/s
- 6.3 rad/s

6. Two of the tires on a farmer’s tractor have diameters of 1.5 m. If the farmer drives the tractor at a linear velocity of 3.0 m/s, what is the angular velocity of each tire?

- 2.0 rad/s
- 4.0 rad/s
- 2.3 rad/s
- 4.5 rad/s

Extended Answer

7. You use a 25-cm long wrench to remove the lug nuts on a car wheel, as shown in the illustration below. If you pull up on the end of the wrench with a force of 2.0 \( \times 10^2 \) N at an angle of 30°, what is the torque on the wrench? (\( \sin 30° = 0.5 \), \( \cos 30° = 0.87 \))

\[ \text{Torque} = F \times r \times \sin \theta \]

\[ = 2.0 \times 10^2 \times 0.25 \times 0.5 \]

\[ = 25 \text{ N·m} \]

Test-Taking TIP

When Eliminating, Cross It Out

Consider each answer choice individually and cross out the ones you have eliminated. If you cannot write in the test booklet, use the scratch paper to list and cross off the answer choices. You will save time and stop yourself from choosing an answer you have mentally eliminated.